NEET | IIT-JEE | FOUNDATION

JEE Advanced - 2021
Paper-I

(Held on 3rd October, 2021)

TEST PAPER
with
SOLUTIONS

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PART A : PHYSICS

SECTION-1 : (Maximum Marks : 12)

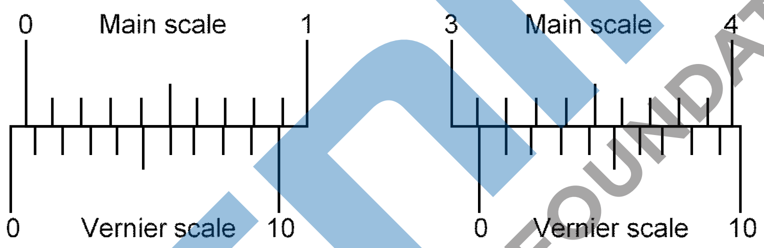
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

1. The smallest division on the main scale of a Vernier calipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calipers with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is



(A) 3.07 cm (B) 3.11 cm (C) 3.15 cm (D) 3.17 cm

Ans. (C)

Sol. Given 10 VSD = 9 MSD

Here MSD $\rightarrow$ Main Scale division

$1\text{ VSD} = \frac{9}{10}\text{ MSD}$

VSD $\rightarrow$ Vernier Scale division

Least count = 1 MSD – 1 VSD

$= \left(1-\frac{9}{10}\right)\text{MSD}$

= 0.1 MSD

= 0.1 × 1.1 cm

= 0.01 cm

As '0' of V.S. lie before '0' of M.S.

Zero error = –[10 – 6]L.C.

= – 4 × 0.01 cm

= –0.04 cm

Reading = 3.1 cm + 1 × LC

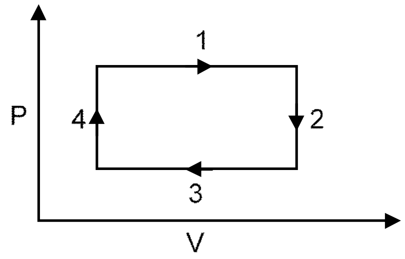
= 3.4 cm + 1 × 0.01 cm

= 3.11 cm

True diameter = Reading – Zero error

= 3.11 – (–0.04) cm = 3.15 cm

2. An ideal gas undergoes a four step cycle as shown in the P – V diagram below. During this cycle, heat is absorbed by the gas in



(A) steps 1 and 2 (B) steps 1 and 3 (C) steps 1 and 4 (D) steps 2 and 4

Ans. (C)

Sol. **Process 1**

P = constant, Volume increases and temperature also increases

$\Rightarrow$ W = positive , ΔU = positive

$\Rightarrow$ Heat is positive and supplied to gas

Process -2

V = constant, Pressure decrease

$\Rightarrow$ Temperature decreases

$W = \int pdV = 0$

ΔT is negative and $\Delta U = \frac{f}{2}nR\Delta T$

$\Rightarrow$ ΔU in negative

$\Delta Q = \Delta U + W$

$\therefore$ $\Delta Q \rightarrow$ Heat is negative and rejected by gas

Process 3

P = constant, Volume decreases

$\Rightarrow$ Temperature also decreases

$W = P\Delta V$ = negative

$\Delta U = \frac{f}{2}nR\Delta T$ = negative

$\Delta Q = W + \Delta U$ = negative

Heat is negative and rejected by gas.

Process 4

V = constant, Pressure increases

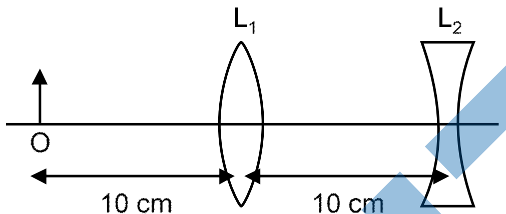
$PV = nRT \Rightarrow$ Temperature increase

$\Rightarrow \quad \Delta U = \frac{f}{2} nR\Delta T$ is positive

$\Delta Q = \Delta U + W$

= positive

3. An extended object is placed at point O, 10 cm in front of a convex lens L_1 and a concave lens L_2 is placed 10 cm behind it, as shown in the figure. The radii of curvature of all the curved surfaces in both the lenses are 20 cm. The refractive index of both the lenses is 1.5. The total magnification of this lens system is



(A) 0.4 (B) 0.8 (C) 1.3 (D) 1.6

Ans. (B)

Sol. Focal length of convex lens (f_1)

$$\frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= (1.5 - 1)\left[\frac{1}{20} - \left(\frac{1}{-20}\right)\right]$$

$$\frac{1}{f_1} = \frac{1}{20}$$

$\Rightarrow \quad \boxed{f_1 = +20\text{cm}}$

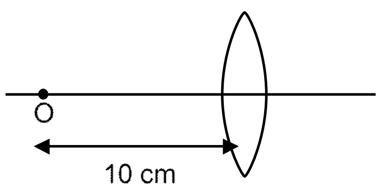
Focal length of concave lens (f_2)

$$\frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f_2} = (1.5 - 1)\left[-\frac{1}{20} - \frac{1}{20}\right] = \frac{1}{-20}$$

$\Rightarrow \quad \boxed{f_2 = -20\text{cm}}$

For lens 1

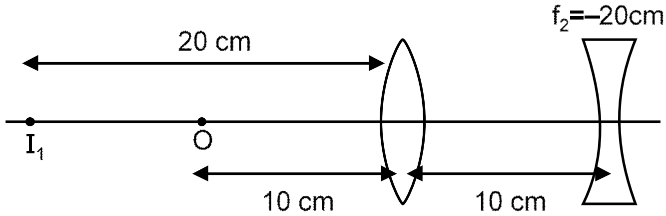


$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$\Rightarrow$ $\boxed{v=-20\text{cm}}$

$$m_1=\frac{v}{u}=\frac{-20}{-10}=2$$

For lens 2



$$u=-30, f=-20, \frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$\boxed{v=-12\text{cm}}$

$$m_2=\frac{v}{u}=\frac{-12}{-30}=\frac{2}{5}$$

Net magnification

$$m=m_1m_2=2\times\frac{2}{5}=\frac{4}{5}=0.8$$

4. A heavy nucleus Q of half-life 20 minutes undergoes alpha-decay with probability of 60% and beta-decay with probability of 40%. Initially, the number of Q nuclei is 1000. The number of alpha decays of Q in the first one hour is

(A) 50 (B) 75 (C) 350 (D) 525

Ans. (D)

Sol. Out of 1000 nuclei of Q 60% may go α-decay

$\Rightarrow$ 600 nuclei may have α-decay

$$\lambda=\frac{\ln 2}{t_{1/2}}=\frac{\ln 2}{20}$$

t = 1 hour = 60 minutes

Using

$$N=N_0e^{-\lambda t}$$

$$=600\times e^{\frac{\ln 2}{20}\times 60}$$

N = 75

$\Rightarrow$ 75 Nuclei are left after one hour

So, no. of nuclei decayed

= 600 – 75 = 525

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;

Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 5 and 6

Question Stem

A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}\,m/s$. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity g = 10 m/s^2.

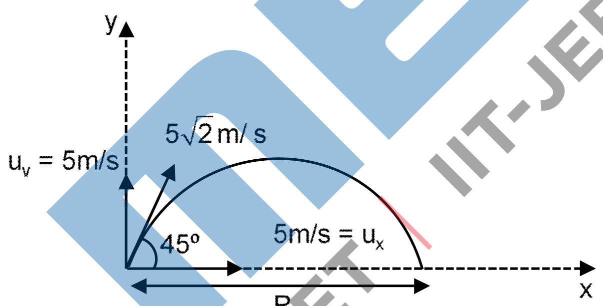
5. The value of t is _______ .

Ans. (0.50)

6. The value of x is _______ .

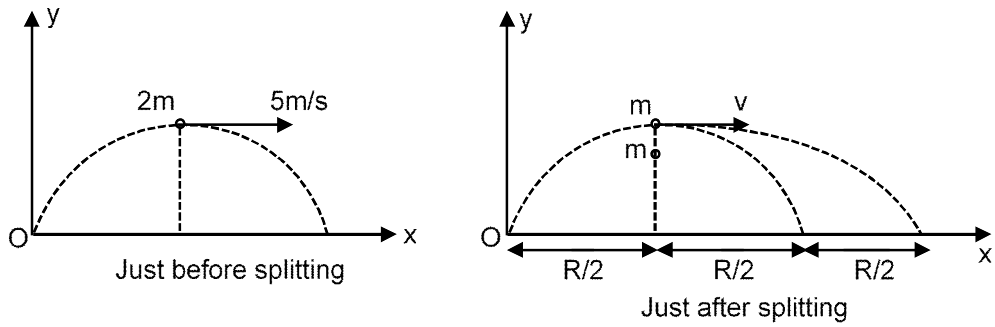
Ans. (7.50)

Sol.



Range $R = \frac{2u_xu_y}{g} = \frac{2\times5\times5}{10} = 5m$

Time of flight $T = \frac{2u_y}{g} = \frac{2\times5}{10} = 1sec$



$\because$ Time of motion of one part falling vertically downwards is = 0.5 sec $= \frac{T}{2}$

$\Rightarrow$ Time of motion of another part, $t = \frac{T}{2} = 0.5\,\text{sec}$

From momentum conservation $\Rightarrow P_i = P_f$

$2m \times 5 = m \times v$

V = 10 m/s

Displacement of other part in 0.5 sec in horizontal direction $= v\frac{T}{2}$

= 10 × 0.5 = 5 m = R

$\therefore$ Total distance of second part from point 'O' is, $x = \frac{3R}{2} = 3 \times \frac{5}{2}$

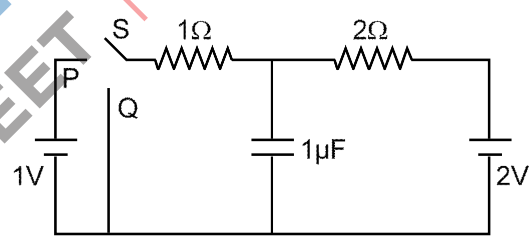
x = 7.5 m

$\Rightarrow$ t = 0.5 sec

Question Stem for Question Nos. 7 and 8

Question Stem

In the circuit shown below, the switch S is connected to position P for a long time so that the charge on the capacitor becomes q_1 μC. Then S is switched to position Q. After a long time, the charge on the capacitor is q_2 μC.



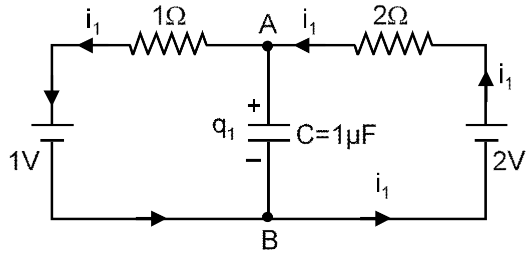
7. The magnitude of q_1 is ___ .

Ans. (1.33)

8. The magnitude of q_2 is ___ .

Ans. (0.67)

Sol.



Switch connected to position 'P'

$V_A - 1 \cdot i_1 - 1 + 2 - 2i_1 = V_A$

$3i_1 = 1$

$i_1 = \frac{1}{3}A$

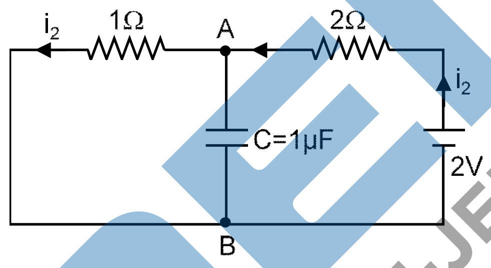
$V_A - 1 \cdot i_1 - 1 = V_B$

Potential drop across capacitor $\Delta V = \frac{4}{3}$volt

$\therefore$ Charge on capacitor $q_1 = C\Delta V$

$= 1 \times \frac{4}{3} \mu C$

$Q_1 = 1.33\ \mu C$



Switch at Position 'Q'

$V_A - 1 \cdot i_2 + 2 - 2i_2 = V_A$

$3i_2 = 2$

$i_2 = \frac{2}{3}A$

$V_2 - i_2 \times 1 = V_B$

$V_A - V_B = i_2 \times 1 = \frac{2}{3}$volt

Potential difference across capacitor $\Delta V = \frac{2}{3}$volt

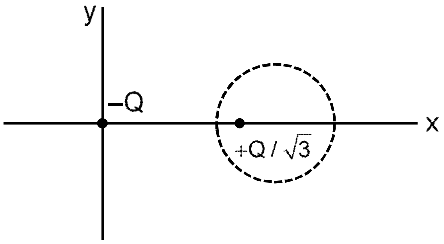
$\therefore$ Charge on capacitor $q_2 = C\Delta V$

$= 1 \times \frac{2}{3} = 0.67\mu C$

Question Stem for Question Nos. 9 and 10

Question Stem

Two point charges –Q and $+Q/\sqrt{3}$ are placed in the xy-plane at the origin (0, 0) and a point (2, 0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential V = 0 in the xy-plane with its center at (b, 0). All lengths are measured in meters.



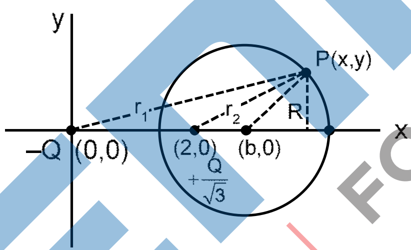
9. The value of R is ___ meter.

Ans. (1.73)

10. The value of b is ___ meter.

Ans. (3.00)

Sol. Let a point P on circle



$$V_P = 0 = \frac{k(-Q)}{r_1} + \frac{kQ/\sqrt{3}}{r_2}$$

$$\frac{kQ}{r_1} = \frac{kQ/\sqrt{3}}{r_2}$$

$$\frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{3}\sqrt{(x-2)^2+y^2}}$$

$3(x-2)^2 + 3y^2 = x^2 + y^2$

$3(x^2 + 4 - 4x) - x^2 + 2y^2 = 0$

$2x^2 + 12 - 12x + 2y^2 = 0$

$x^2 + 6 - 6x + y^2 = 0$

$$(x-3)^2 + y^2 = \left(\sqrt{3}\right)^2$$

$R = \sqrt{3} = 1.73$,

b = 3

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
- Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
- Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
- Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
- Zero Marks : 0 If unanswered;
- Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

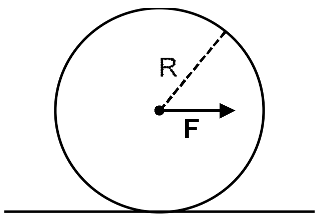
  choosing ONLY (B) will get +1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

  choosing any other option(s) will get −2 marks.

11. A horizontal force F is applied at the center of mass of a cylindrical object of mass m and radius R, perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ. The center of mass of the object has an acceleration a. The acceleration due to gravity is g. Given that the object rolls without slipping, which of the following statement(s) is(are) correct?



(A) For the same F, the value of a does not depend on whether the cylinder is solid or hollow

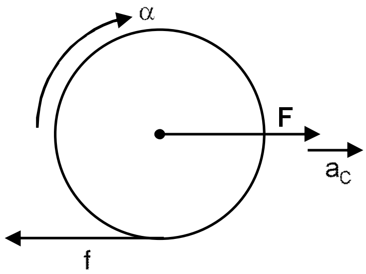
(B) For a solid cylinder, the maximum possible value of a is 2μg

(C) The magnitude of the frictional force on the object due to the ground is always μmg

(D) For a thin-walled hollow cylinder, $a = \frac{F}{2m}$

Ans. (BD)

Sol.



$F - f = ma_C$

$fR = I_C\alpha$

$a_C - \alpha R = 0$

$$F - I_C\frac{\alpha}{R} = ma_C$$

$$a_C = \frac{F}{\frac{I_C}{R^2} + m}$$

$$f = \frac{I_C\alpha}{R} = \frac{I_C}{R^2}a_C = \frac{I_C}{R^2}\frac{F}{\left[\frac{I_C}{R^2} + m\right]}$$

$$f = \frac{F}{\left(m + \frac{I_C}{R^2}\right)}$$

Thin walled hollow cylinder

$I_C = mR^2$

$$a_C = \frac{F}{2m}$$

$$fR = I_C\alpha = \frac{I_C a_C}{R}$$

$$f = \frac{I_C a_C}{R^2} \le \mu mg$$

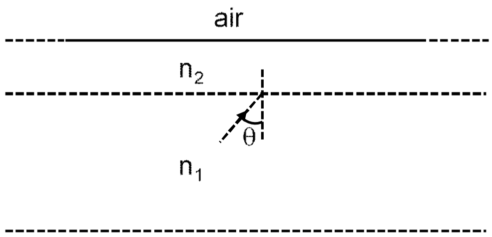
$$a_C \le \frac{\mu mgR^2}{I_C}$$

for solid cylinder $I_C = \frac{mR^2}{2}$

$a_C \le 2\mu g$

$(a_C)_{max} = 2\mu g$

12. A wide slab consisting of two media of refractive indices n_1 and n_2 is placed in air as shown in the figure. A ray of light is incident from medium n_1 to n_2 at an angle θ, where $\sin\theta$ is slightly larger than $1/n_1$. Take



(A) The light ray enters air if $n_2 = n_1$

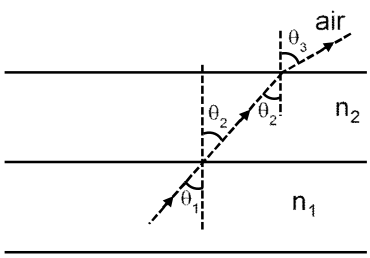
(B) The light ray is finally reflected back into the medium of refractive index n1 if $n_2 < n_1$

(C) The light ray is finally reflected back into the medium of refractive index n1 if $n_2 > n_1$

(D) The light ray is reflected back into the medium of refractive index n_1 if $n_2 = 1$

Ans. (BCD)

Sol.



$\sin\theta > \frac{1}{n_1}$ (Given)

i.e. $\sin\theta_1 > \frac{1}{n_1}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$

If $n_1 = n_2$ then $\theta_2 = \theta_1$

$n_2 \sin\theta_2 = (1) \sin\theta_3$

$\sin\theta_3 = n_2 \sin\theta_2$

$\sin\theta_3 = n_1 \sin\theta_1$

$\sin\theta_1 = \frac{\sin\theta_3}{n_1} > \frac{1}{n_1}$

$\sin\theta_3 > 1$

$\theta_3 > 90^\circ$

This means ray cannot enter air

For $n_1 > n_2$; $\sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2 > \frac{1}{n_1}$

$\sin\theta_2 > \frac{1}{n_2}$

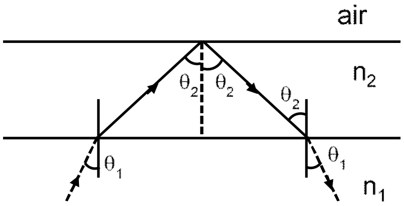
for surface 2 – air interface

$n_2 \sin\theta_2 = \sin\theta_3$

$$\sin\theta_2 = \frac{\sin\theta_3}{n_2} > \frac{1}{n_2}$$

$\theta_2 > 90^o$

It means ray is reflected back in medium-2



for surface 1 – surface 2 interface

$n_2 \sin\theta_2 = n_1 \sin\theta_1$

$$\sin\theta_{2C} = \frac{n_1}{n_2}$$

θ_{2C} : critical angle

for any to enter medium-1

$\theta_2 < \theta_{2C}$

$\sin\theta_2 < \sin 2\theta_C$

$$\frac{n_1}{n_2}\sin\theta_1 < \frac{n_1}{n_2}$$

$\sin\theta_1 = 1$

$\theta_1 < 90^o$, which is true

Hence ray enters medium-1

For $n_2 > n_1$

$$\frac{n_2}{n_1}\sin\theta_2 > \frac{n_2}{n_1}$$

$$\sin\theta_2 > \frac{1}{n_2}$$

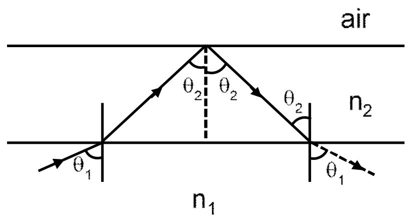
For surface 2 – air interface

$n_2 \sin\theta_2 = \sin\theta_3$

$$\sin\theta_2 = \frac{\sin\theta_3}{n_2} > \frac{1}{n_2}$$

$\theta_2 > 90$

It means ray is reflected back in medium – 2



$n_2 \sin\theta_2 = n_1 \sin\theta_1$

$$\sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2$$

$$\sin\theta_{2C} = \frac{n_1}{n_2}; \theta_{2C} \rightarrow \text{critical angle}$$

For any to enter medium – 1

$\theta_2 < \theta_{2C}$

$\sin\theta_2 < \sin\theta_{2C}$

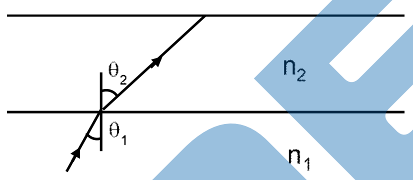
$$\frac{n_1}{n_2}\sin\theta_1 < \frac{n_1}{n_2}$$

$\sin\theta_1 < 1$

$\theta_1 < 90^\circ$, which is true

Hence ray enters medium – 1

Let $n_2 = 1$



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_2 = 1$

$n_1 \sin\theta_1 = \sin\theta_2$

$$\sin\theta_1 = \frac{\sin\theta_2}{n_1} > \frac{1}{n_1}$$

$\sin\theta_2 > 1 \Rightarrow \theta_2 > 90^\circ$

ray is reflected back in medium –

13. A particle of mass M = 0.2 kg is initially at rest in the xy-plane at a point ($x = -\ell$, $y = -h$), where $\ell = 10$ m and h = 1m. The particle is accelerated at time t = 0 with a constant acceleration a = 10 m/s^2 along the positive x-direction. Its angular momentum and torque with respect to the origin, in SI units, are represented by $\vec{L}$ and $\vec{\tau}$ respectively. $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the positive x, y and z-directions, respectively. If $\hat{k} = \hat{i} \times \hat{j}$ then which of the following statement(s) is(are) correct ?

(A) The particle arrives at the point ($x = \ell$, $y = -h$) at time t = 2s.

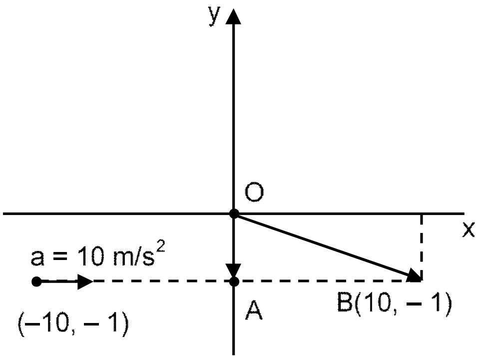
(B) $\vec{\tau} = 2\hat{k}$ when the particle passes through the point ($x = \ell$, $y = -h$)

(C) $\vec{L} = 4\hat{k}$ when the particle passes through the point ($x = \ell$, $y = -h$)

(D) $\vec{\tau} = \hat{k}$ when the particle passes through the point ($x = 0$, $y = -h$)

Ans. (ABC)

Sol.



$\vec{r}_A = -\hat{j}$

$S = \frac{1}{2}at^2$

$20 = \frac{1}{2} \times 10 \times t^2$

t = 2 sec

$\vec{\tau}_0 = \vec{r} \times \vec{F}$; $\vec{r}_B = 10\hat{i} - \hat{j}$

$\vec{F} = m\vec{a} = 0.2 \times 10\hat{i} = 2\hat{i}$

$\vec{\tau}_0 = \left(10\hat{i} - \hat{j}\right) \times \left(2\hat{i}\right)$

$\vec{\tau}_0 = 2\hat{k}$

$\vec{L}_0 = \vec{r}_n \times \vec{p} = \vec{r}_B \times m\vec{v}$

$\vec{v} = \vec{a}t = 10\hat{i} \times 2 = 20\hat{i}$

$\vec{L}_0 = (0.2)\left[\left(10\hat{i} - \hat{j}\right) \times 20\hat{i}\right] = 4\hat{k}$

At point A(0, –1)

$\vec{\tau}_0 = \vec{r}_A \times \vec{F} = \left(-\hat{j}\right) \times 2\hat{i} = 2\hat{k}$

14. Which of the following statement(s) is(are) correct about the spectrum of hydrogen atom ?

(A) The ratio of the longest wavelength to the shortest wavelength in Balmer series is 9/5

(B) There is an overlap between the wavelength ranges of Balmer and Paschen series.

(C) The wavelengths of Lyman series are given by $\left(1+\frac{1}{m^2}\right)\lambda_0$, where λ_0 is the shortest wavelength of Lyman series and m is an integer

(D) The wavelength ranges of Lyman and Balmer series do not overlap

Ans. (AD)

Sol. For A

When the transition is from any level to n = 2, then photon emitted belong to Balmer series.

$\therefore$ For longest wavelength, transition occurs from n = 3 to n = 2.

$\therefore \frac{hc}{\lambda_{max}} = RCh\left[\frac{1}{2^2}-\frac{1}{3^2}\right]$ & for shortest wavelength transition occurs from n = ∞ to n = 2

$$\therefore \frac{hc}{\lambda_{min}} = RCh\left[\frac{1}{2^2}-\frac{1}{\infty^2}\right]$$

$$\therefore \frac{\lambda_{longest}}{\lambda_{shorts}} = \frac{9}{5}$$

For (B)

$$\lambda_{longest} \text{ of Balmer} = \frac{36}{5R}$$

$$\lambda_{shorts} \text{ of Paschen} = \frac{9}{5R}$$

Hence these wavelength don't overlap.

For (C)

For Lyman series,

$$\frac{1}{\lambda} = R\left[\frac{1}{1}-\frac{1}{m^2}\right]$$

Also $\frac{1}{\lambda_0} = R$

$$\frac{1}{\lambda} = \frac{1}{\lambda_0}\left[1-\frac{1}{m^2}\right] \Rightarrow \lambda = \frac{\lambda_0}{1-\frac{1}{m^2}}$$

For (D)

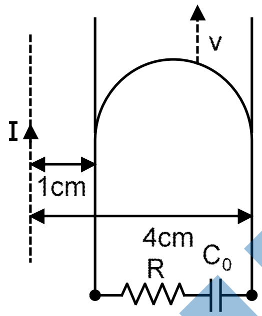
$$\lambda_{longest} \text{ of Lyman} = \frac{4}{3R},\ \lambda_{shortest} \text{ of Balmer} = \frac{4}{R}$$

Hence that wavelength don't overlap.

15. A long straight wire carries a current, I = 2 ampere. A semi-circular conducting rod is placed beside it on two conducting parallel rails of negligible resistance. Both the rails are parallel to the wire. The wire, the rod and the rails lie in the same horizontal plane, as shown in the figure. Two ends of the semi-circular rod are at distances 1cm and 4 cm from the wire. At time t = 0, the rod starts moving on the rails with a speed v = 3.0 m/s (see the figure).

A resistor R = 1.4 Ω and a capacitor C_0 = 5.0 μF are connected in series between the rails. At time t = 0, C0 is uncharged. Which of the following statement(s) is(are) correct ?

[$\mu_0 = 4\pi \times 10^{-7}$ SI units. Take ln 2 = 0.7]



(A) Maximum current through R is 1.2×10^{-6} ampere

(B) Maximum current through R is 3.8×10^{-6} ampere

(C) Maximum charge on capacitor C_0 is 8.4×10^{-12} coulomb

(D) Maximum charge on capacitor C_0 is 2.4×10^{-12} coulomb

Ans. (AC)

Sol. EMF developed across the emf of semi-circular rod =

$$\int_1^4 \frac{\mu_0 i}{2\pi r} drv = \frac{\mu_0 iv}{2\pi}\ell n4 = \frac{\mu_0 iv}{\pi}\ell n2$$

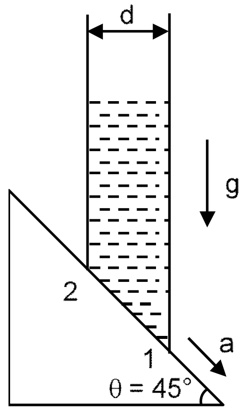
Form given value,

$$E = \frac{4\pi \times 10^{-7} \times 2 \times 3 \times 0.7}{\pi} = 24 \times 7 \times 10^{-8}$$

$$i_{max} = \frac{E}{R} = \frac{24 \times 7 \times 10^{-8}}{1.4} = 1.2 \times 10^{-6}$$

$Q_{max} = C_0E = 24 \times 7 \times 10^{-8} \times 5 \times 10^{-6} = 8.4 \times 10^{-12}$ C

16. A cylindrical tube, with its base as shown in the figure, is filled with water. It is moving down with a constant acceleration a along a fixed inclined plane with angle θ = 45°. P1 and P2 are pressures at points 1 and 2, respectively, located at the base of the tube. Let $\beta = (P_1 - P_2)/(\rho g d)$, where ρ is density of water, d is the inner diameter of the tube and g is the acceleration due to gravity. Which of the following statement(s) is(are) correct ?



(A) $\beta = 0$ when $a = g/\sqrt{2}$

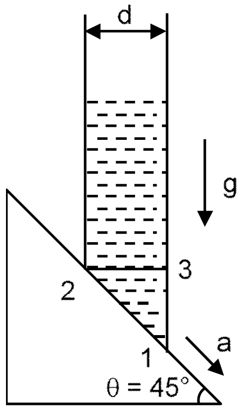
(B) $\beta > 0$ when $a = g/\sqrt{2}$

(C) $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when a = g/2

(D) $\beta = \frac{1}{\sqrt{2}}$ when a = g/2

Ans. (AC)

Sol.



$$\therefore P_1 - P_3 = \rho\left(g - \frac{a}{\sqrt{2}}\right)d$$

$$P_2 - P_3 = \rho\frac{a}{\sqrt{2}}d$$

$$\therefore P_1 - P_2 = \rho d\left[g - \frac{2a}{\sqrt{2}}\right]$$

$$\therefore \frac{P_1 - P_2}{\rho g d} = \left[1 - \sqrt{2}\frac{a}{g}\right] = \beta$$

if $\beta = 0$, $a = \frac{g}{\sqrt{2}}$ ..(A)

$\beta = \frac{\sqrt{2}-1}{2}$, $a = \frac{g}{\sqrt{2}}$ ....(C)

SECTION-4 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

17. An α-particle (mass 4 amu) and a singly charged sulfur ion (mass 32 amu) are initially at rest. They are accelerated through a potential V and then allowed to pass into a region of uniform magnetic field which is normal to the velocities of the particles. Within this region, the α-particle and the sulfur ion move in circular orbits of radii r_α and r_s, respectively. The ratio (r_s/r_α) is___.

Ans. (4)

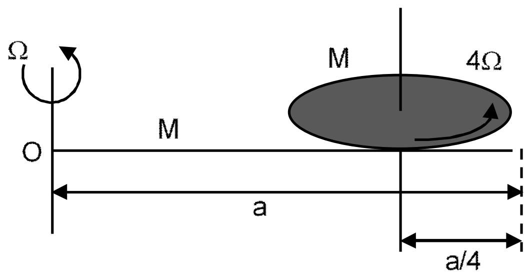
Sol. $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2mqV}}{qB}$

$$\frac{P^2}{2m} = K.E. = qV$$

$$\frac{r_s}{r_\alpha} = \sqrt{\frac{32}{1} \times \frac{2}{4}} = 4$$

$$\frac{r_s}{r_\alpha} = 4$$

18. A thin rod of mass M and length a is free to rotate in horizontal plane about a fixed vertical axis passing through point O. A thin circular disc of mass M and of radius a/4 is pivoted on this rod with its center at a distance a/4 from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity □ and the disc rotating about its vertical axis with angular velocity 4Ω. The total angular momentum of the system about the point O is $\left(\dfrac{ma^2\Omega}{48}\right)n$. The value of n is___.



Ans. (49)

Sol. $L = \frac{Ma^2}{3}\Omega + M\left(\frac{3a}{4}\right)\Omega + \frac{M\left(\frac{a}{4}\right)^2 4\Omega}{2}$

$L = \frac{49}{48}Ma^2\Omega$

n = 49

19. A small object is placed at the center of a large evacuated hollow spherical container. Assume that the container is maintained at 0 K. At time t = 0, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t = t_1$ and 50 K at $t = t_2$. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio (t_2/t_1) is____.

Ans. (9)

Sol. $\sigma AT^4 = -ms\frac{dT}{dt}$

$\int_{200}^{100}\frac{dT}{T^4} = \int_0^{t_1} kdt$

$\left.\frac{1}{3T^3}\right|_{200}^{100} = kt_1$

$\frac{1}{3}\left(\frac{1}{100^3} - \frac{1}{200^3}\right) = kt_1$

$\frac{t_2}{t_1} = \left(\frac{200^3 - 50^3}{200^3 - 100^3}\right)\frac{100^3}{50^3} = 9$

PART B : CHEMISTRY

SECTION-1 : (Maximum Marks : 12)

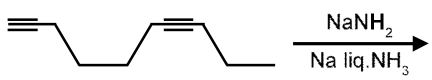
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

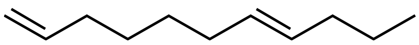
Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

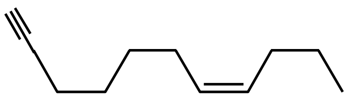
Negative Marks : −1 In all other cases.

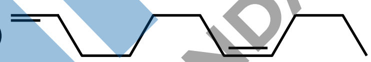
1. The major product formed in the following reaction is



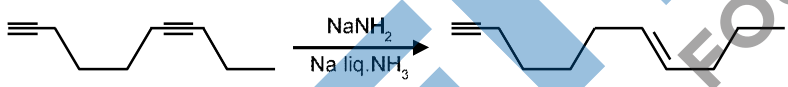
(A) 

(B) 

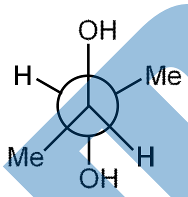
(C) 

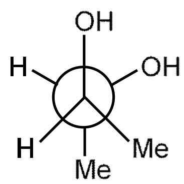
(D) 

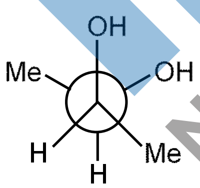
Ans. (B)

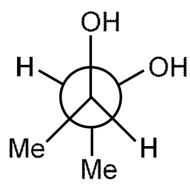
Sol. 

2. Among the following, the conformation that corresponds to the most stable conformation of meso–butane–2,3–diol is –

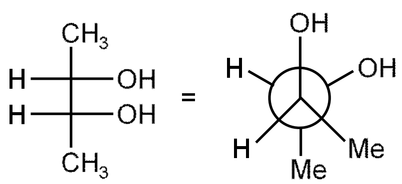
(A) 

(B) 

(C) 

(D) 

Ans. (B)

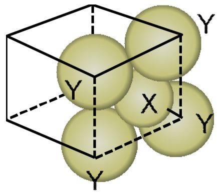
Sol. 

Most stable conformation with H-bonding

Meso butane –2,3,–diol

3. For the given close packed structure of a salt made of cation X and anion Y shown below (ions of only one face are shown for clarity) , the packing fraction is approximately

$\left(\text{packing fraction} = \dfrac{\text{Packing efficiency}}{100}\right)$



(A) 0.74 (B) 0.63 (C) 0.52 (D) 0.48

Ans. (B)

Sol. Packing fraction (P.F.) $= \dfrac{1 \times \frac{4}{3}\pi r_-^3 + 3 \times \frac{4}{3}\pi r_+^3}{a^3}$

$\dfrac{r_+}{r_-} = 0.414$ (square planar void), a = 2r_

We get,

$$\text{P.F.} = \frac{\frac{4}{3}\pi\left(r_-^3 + 3r_+^3\right)}{8r_-^3}$$

$$= \left[\frac{\pi}{6}\left(1 + 3(0.414)^3\right)\right]$$

= 0.63

4. The calculated spin only magnetic moments of $[Cr(NH_3)_6]^{3+}$ and $[CuF_6]^{3-}$ in BM, respectively, are (Atomic numbers of Cr and Cu are 24 and 29, respectively)

(A) 3.87 and 2.84 (B) 4.90 and 1.73 (C) 3.87 and 1.73 (D) 4.90 and 2.84

Ans. (A)

Sol. $[Cr(NH_3)_6]^{3+}$

$Cr^{3+} \Rightarrow [Ar]3d^3$

In presence of NH_3 ligand

		eg
1	1	1	t_{2g}

Number of unpaired electrons = 3

$\mu = \sqrt{n(n+2)}$ B.M.

$\mu = \sqrt{3(3+2)}$ B.M.

$\mu = \sqrt{15}$ B.M.

$\Rightarrow$ 3.87 B.M.

$[CuF_6]^{3-}$

$Cu^{3+} \Rightarrow [Ar]3d^8$

In presence of F^- Ligand

$Cu^{3+} \Rightarrow$ $\boxed{1}\boxed{1}$ eg

$\boxed{1\!\downharpoonleft}\boxed{1\!\downharpoonleft}\boxed{1\!\downharpoonleft}$ t_{2g}

Number of unpaired electrons = 2

$\mu = \sqrt{n(n+2)}$ B.M.

$\mu = \sqrt{2(2+2)} \Rightarrow \sqrt{8}$ B.M.

$\Rightarrow 2.84$ B.M

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

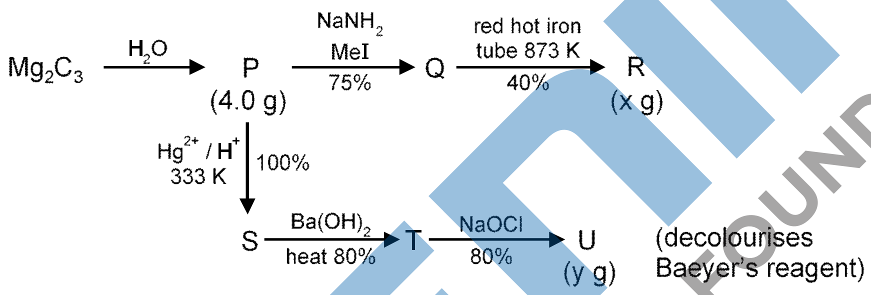
Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;

Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 5 and 6

Question stem

For the following reaction scheme, percentage yields are given along the arrow :



x g and y g are mass of R and U, respectively.

(Use : Molar mass (in g mol^{-1}) of H, C and O as 1, 12 and 16, respectively)

5. The value of x is_____.

Ans. (1.62)

Sol. $Mg_2C_3 + 4H_2O \rightarrow 2Mg(OH)_2 + CH_3C \equiv CH$

$$CH_3 \equiv CH \xrightarrow{NaMe_2} CH_3 - C \equiv C^-Na^+$$

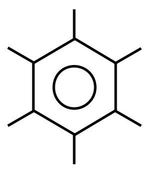
$$\downarrow MeI$$

$$CH_3 - C \equiv C - CH_3 \text{ (0.075 mmol)}$$

(Q)

$$3CH_3 - C \equiv C - CH_3 \xrightarrow[\text{iron tube } 873\text{ K}]{\text{red hot}}$$

0.75 mole



(0.075 × 0.4)

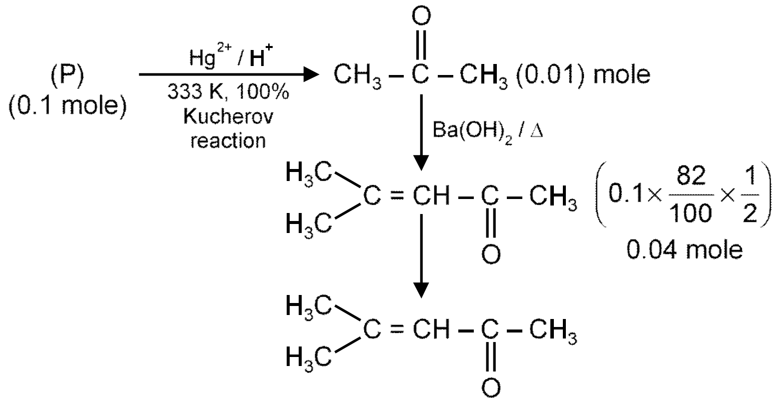
= 0.01 mole

The value of x = 162 × 0.01 = 1.62 gm

6. The value of y is______.

Ans. (3.2)

Sol.



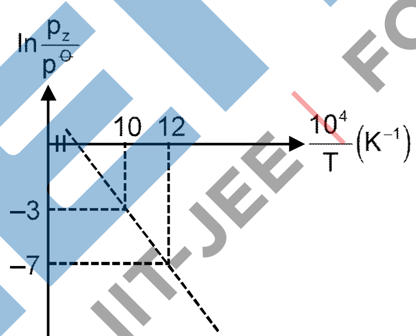
$\left(0.04\times\frac{80}{100}\right)=0.032$ mole

60 + 32 + 8 = 100

The value of Y = 0.032 × 100 = 3.2

Question Stem for Question Nos. 7 and 8

For the reaction X(s) ⇌ Y(s) + Z(g), the plot of $\ln\frac{p_z}{p^{\ominus}}$ versus $\frac{10^4}{T}$ is given below (in solid line), where p_z is the pressure (in bar) of the gas Z at temperature T and $P^{\ominus}=1$ bar.



(Given, $\frac{d(\ln K)}{d\left(\frac{1}{T}\right)}=-\frac{\Delta H^{\ominus}}{R}$ where the equilibrium constant, $K=\frac{p_z}{p^{\ominus}}$ and the gas constant, R = 8.314 J K^{-1} mol^{-1})

7. The value of standard enthalpy, $\Delta H^{\ominus}$ (in kJ mol^{-1}) for the reaction is_____.

Ans. (166.28)

Sol. $\Delta G^\circ=-RT\ln\left(\frac{P}{1}\right)=\Delta H^\circ-T\Delta S^\circ$

$$\ln\left(\frac{P}{1}\right)=-\frac{\Delta H^\circ}{RT}+\frac{\Delta S^\circ}{R}$$

$$\text{Slope}=-\frac{\Delta H^\circ}{RT}=10^4\times\left(-\frac{4}{2}\right)$$

$\Rightarrow \Delta H° = 2 \times 10^4 \times R$

$= 166.28$ kJ/mole

8. The value of $\Delta S^{\ominus}$ (in J K^{-1} mol^{-1}) for the given reaction, at 1000 K is______ .

Ans. (141.33 or 141.34)

Sol. From the plot when, $\frac{10^4}{T} = 10 \quad \Rightarrow \quad T = 1000$ K

$$\ln\left(\frac{P_2}{1}\right) = -3$$

Substituting in equation:

$$\ln\left(\frac{P_2}{1}\right) = -\frac{\Delta H°}{RT} + \frac{\Delta S°}{R}$$

We get,

$$-3 = -\frac{2 \times 10^4 \times R}{R \times 1000} + \frac{\Delta S°}{R}$$

$\Rightarrow \Delta S° = 17R$

$\Rightarrow \Delta S° = 17 \times 8.314$ J/K-mol

$\Rightarrow \Delta S° = 141.34$ J/K-mol

Question Stem for Question Nos. 9 and 10

The boiling point of water in a 0.1 molal silver nitrate solution (solution A) is x ºC. To this solution A, an equal volume of 0.1 molal aqueous barium chloride solution is added to make a new solution B. The difference in the boiling points of water in the two solutions A and B is y × 10^{-2} ºC.

(Assume : Densities of the solutions A and B are the same as that of water and the soluble salts dissociate completely.)

Use: Molal elevation constant (Ebullioscopy Constant), K_b = 0.5 K kg mol^{-1}; Boiling point of pure water as 100ºC.)

9. The value of x is _______.

Ans. (100.10)

Sol. $AgNO_3(aq) \longrightarrow Ag^+(aq) + NO_3^-(aq)$

0.1 m   0.1 m

$\Delta T_b = 0.2 \times 0.5$

$= 0.1°C = 0.1$ K

Boiling point of solution = 100.1°C

= X

10. The value of y is _____ .

Ans. (2.50)

Sol. $AgNO_3(aq) \longrightarrow Ag^+(aq) + NO_3^-(aq)$

0.05 m 0.05 m 0.05 m

$BaCl_2(aq) \longrightarrow Ba^{2+}(aq) + 2Cl^-(aq)$

0.05 m 0.05 m 0.1 m

Ag^+ and Cl^- combine to form AgCl precipitate

$$Ag^+(aq) + Cl^-(aq) \longrightarrow AgCl(s)$$

t = 0 0.05 m 0.1 m

t = ∞ 0 0.05 m

In final solution total concentration of all ions :

$[Cl^-] + [NO_3^-] + [Ba^{2+}] = 0.05 + 0.05 + 0.05$

$= 0.15$ m

$\Delta T_b = 0.5 \times 0.15$

$= 0.075\ °C$

B.P. of solution 'B' = 100.075°C

B.P. of solution 'A' = 100.1°C

$|y| = 100.1 - 100.075$

$= 0.025 = 2.5 \times 10^{-2}$

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
- Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
- Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
- Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
- Zero Marks : 0 If unanswered;
- Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

  choosing ONLY (B) will get +1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

  choosing any other option(s) will get −2 marks.

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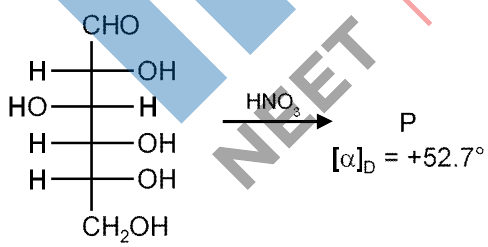
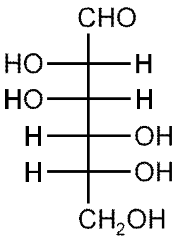
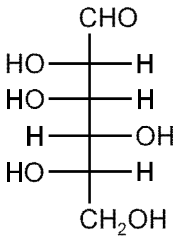
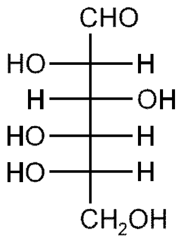
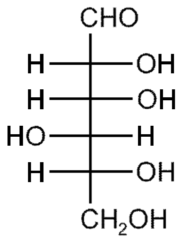
11. Given



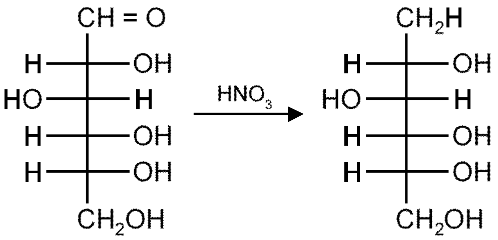
D-Glucose

The compound(s), which on reaction with HNO_3 will give the product having degree of rotation, $[\alpha]_D = -52.7^o$ is (are)

(A)  (B)  (C)  (D) 

Ans. (C,D)

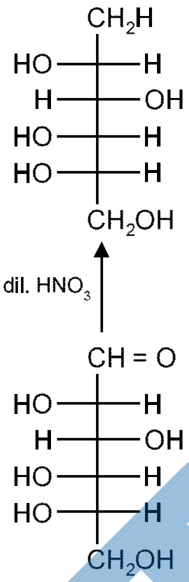
Sol.



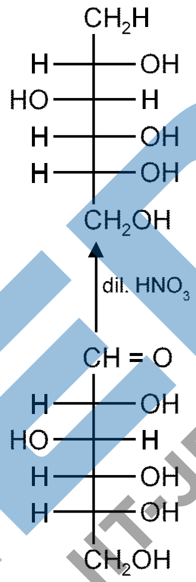
(P)

$[\alpha]_D = 52.7°$

The enantiomer of P has rotation –52.7° is as follows

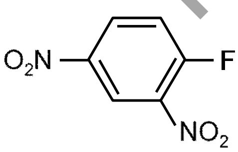


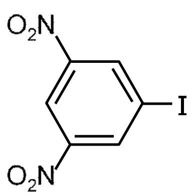
(C)

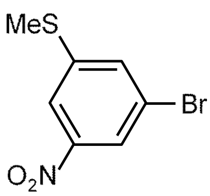


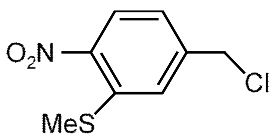
(D)

12. The reaction of Q with PhSNa yields an organic compound (major product) that gives positive Carius test on treatment with Na_2O_2 followed by addition of $BaCl_2$. The correct option(s) for Q is (are).

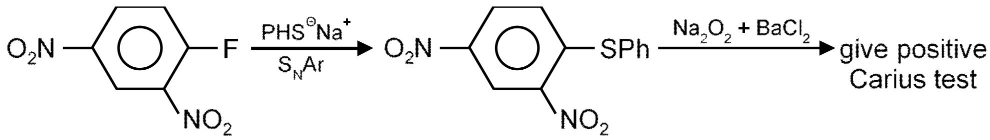
(A) 

(B) 

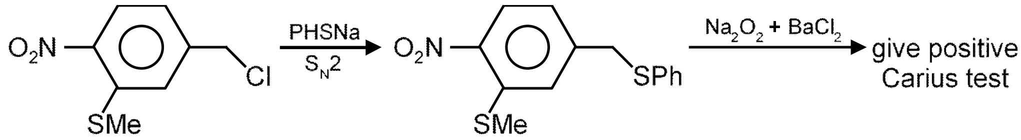
(C) 

(D) 

Sol. (A)



(D)



13. The correct statement(s) related to colloids is(are)

(A) The process of precipitating colloidal sol by an electrolyte is called peptization.

(B) Colloidal solution freezes at higher temperature than the true solution at the same concentration.

(C) Surfactants form micelle above critical micelle concentration (CMC). CMC depends on temperature

(D) Micelles are macromolecular colloids.

Ans. (B,C)

Sol. (A) Process of precipitating colloidal solution is called coagulation. Hence false.

(B) For colloidal solutions concentration is very small due to very large molar mass and hence their colligative properties are very small as compared to true solutions

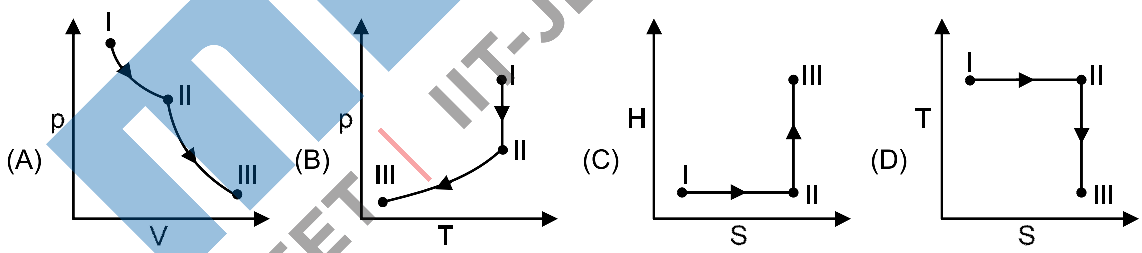
$\therefore$ ΔT_f is lesser for colloidal solution. Hence true.

(C) At CMC surfactant form micelles. Hence true

(D) Micelles and macromolecular colloids are two different types of colloids. Hence false.

14. An ideal gas undergoes a reversible isothermal expansion from state I to state II followed by a reversible adiabatic expansion from state II to state III. The correct plot(s) representing the changes from state I to state III is(are)

(p : pressure, V : volume, T : temperature, H : enthalpy, S : entropy)



Ans. (A,B,D)

Sol. From state I to II (Reversible isothermal expansion)

$\Rightarrow$ P decreases, V increases, T constant

H constant & S increases.

From state II to III (Reversible adiabatic expansion)

$\Rightarrow$ P decreases, V increases, T decreases

H decreases, S constant

$\therefore$ Plots (A), (B), (D) are correct while (C) is wrong as from II to III, H is decreasing.

15. The correct statement(s) related to the metal extraction processes is(are)

(A) A mixture of PbS and PbO undergoes self-reduction to produce Pb and SO_2.

(B) In the extraction process of copper from copper pyrites, silica is added to produce copper silicate.

(C) Partial oxidation of sulphide ore of copper by roasting, followed by self-reduction produces blister copper.

(D) In cyanide process, zinc powder is utilized to precipitate gold from $Na[Au(CN)_2]$

Ans. (A,C,D)

Sol. (A) $PbS + 2PbO \rightarrow 3Pb + SO_2$ (self reduction)

(B) Silica is added to remove impurity of Fe in the form of slag $FeSiO_3$

(C) $CuFeS_2$ ore is partially oxidized first by roasting and then self reduction of Cu takes place to produce blister copper.

(D) $4Na[\overset{+1}{Au}(CN)_2] + \underset{\text{Reducing Agent}}{2Zn} \longrightarrow 2Na_2[\overset{+2}{Zn}(CN)_4] + 4\overset{0}{Au}$

16. A mixture of two salts is used to prepare a solution S, which gives the following results :

$$\underset{\text{only}}{\text{White precipuitate (s)}} \xleftarrow[\text{Room temperature}]{\text{Dilute NaoH(aq.)}} \underset{\text{(aq. solution of the salts)}}{S} \xrightarrow[\text{Room temperature}]{\text{Dilute NaoH(aq.)}} \underset{\text{only}}{\text{White Precipuitate (s)}}$$

The correct option(s) for the salt mixture is(are)

(A) $Pb(NO_3)_2$ and $Zn(NO_3)_2$

(B) $Pb(NO_3)_2$ and $Bi(NO_3)_3$

(C) $AgNO_3$ and $Bi(NO_3)_3$

(D) $Pb(NO_3)_2$ and $Hg(NO_3)_2$

Ans. (A,B)

Sol. $Pb(NO_3)_2 \xrightarrow{\text{dil.HCl}} \underset{\text{White ppt.}}{PbCl_2} \downarrow$

$Bi(NO_3)_3 \xrightarrow{\text{dil.HCl}} \underset{\text{Water soluble}}{BICl_3}$

$Hg(NO_3)_2 \xrightarrow{\text{dil.HCl}} \underset{\text{Water soluble}}{HgCl_2}$

$AgNO_3 \xrightarrow{\text{dil.HCl}} \underset{\text{White ppt.}}{AgCl} \downarrow$

$Zn(NO_3)_2 \xrightarrow{\text{dil.HCl}} \underset{\text{Water soluble}}{ZnCl_2}$

$Pb(NO_3)_2 \xrightarrow{\text{NaOH(dil.)}} \underset{\text{White ppt.}}{Pb(OH)_2} \downarrow$

$Zn(NO_3)_2 \xrightarrow{\text{NaOH(dil.)}} \underset{\text{White ppt.}}{Zn(OH)_2} \downarrow$

$Bi(NO_3)_3 \xrightarrow{\text{NaOH(dil.)}} \underset{\text{White ppt.}}{Bi(OH)_3} \downarrow$

$AgNO_3 \xrightarrow{\text{NaOH(dil.)}} \underset{\text{Brown ppt.}}{Ag_2O}$

$Hg(NO_3)_3 \xrightarrow{\text{NaOH(dil.)}} \underset{\text{yellow ppt.}}{HgO} \downarrow$

SECTION-4 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

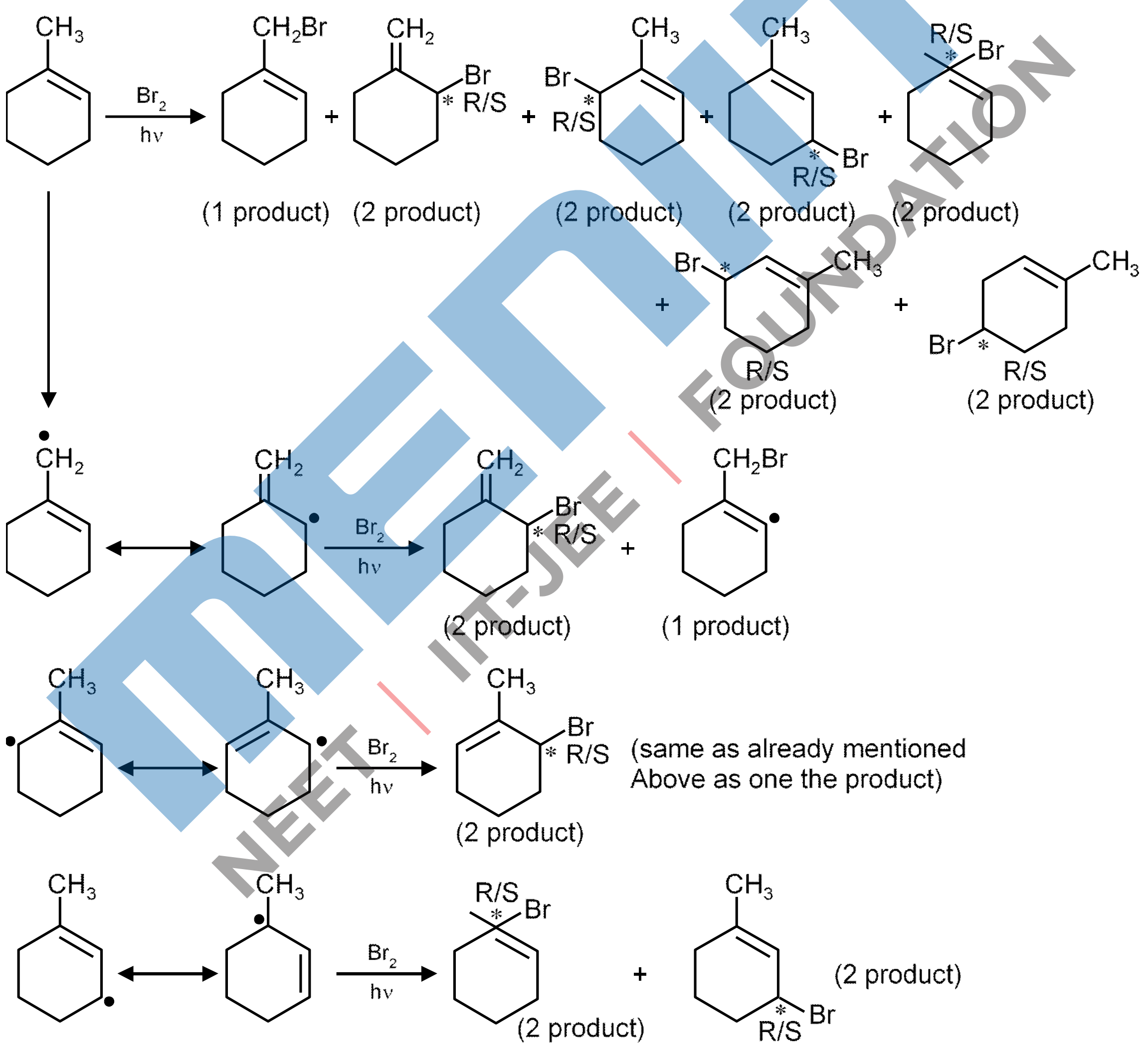
  Full Marks : +4 If ONLY the correct integer is entered;

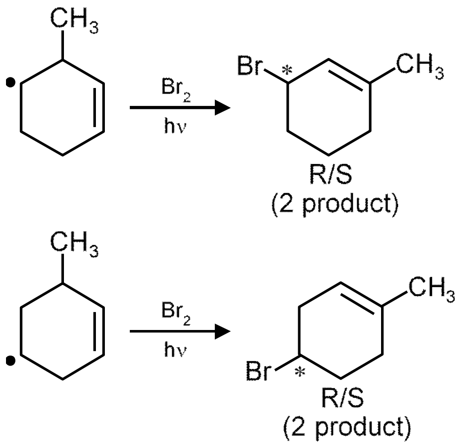
  Zero Marks : 0 In all other cases.

17. The maximum number of possible isomers (including stereoisomers) which may be formed on mono-bromination of 1-methylcyclohex-1-ene using Br_2 and UV light is ____

Ans. (13)

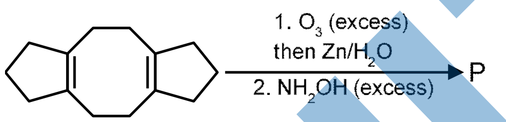
Sol.





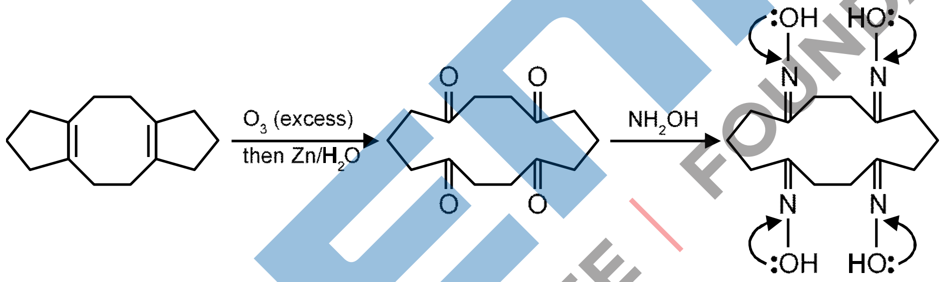
Total 13 product

18. In the reaction given below, the total number of atoms having sp^2 hybridization in the major product P is ____



Ans. (12)

Sol.



Total 12 atoms are sp^2 hybridised

19. The total number of possible isomers for $[Pt(NH_3)_4Cl_2]Br_2$ is ______

Ans. (6)

Sol. Isomers

(I) $[Pt\ (NH_3)_4\ Cl_2]Br_2 \Rightarrow$ G.I. = 2

(II) $[Pt\ (NH_3)_4\ Br_2]\ Cl_2 \Rightarrow$ G.I. = 2

(III) $[Pt\ (NH_3)_4\ BrCl]\ Br.Cl \Rightarrow$ G.I. = 2

I, II, III are ionisation isomers of each other, each having 2 geometrical isomers.

Total possible isomers will be 6

PART C : MATHEMATICS

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

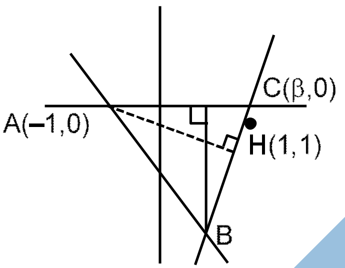
Negative Marks : −1 In all other cases.

1. Consider a triangle Δ whose two sides lie on the x-axis and the line x + y + 1 = 0. If the orthocenter of Δ is (1, 1), then the equation of the circle passing through the vertices of the triangle Δ is

(A) $x^2 + y^2 - 3x + y = 0$ (B) $x^2 + y^2 + x + 3y = 0$ (C) $x^2 + y^2 + 2y - 1 = 0$ (D) $x^2 + y^2 + x + y = 0$

Ans. (B)

Sol.



$(1,-2) = (\alpha,-\alpha-1)$

$\Rightarrow \alpha = 1$

one of the vectex is intersection of x-axis and x + y + 1 = 0 ⇒ A(–1,0)

Let vertex B be $(\alpha,-\alpha-1)$

Line AC ⊥ BH ⇒ $\alpha = 1$ ⇒ B(1,–2)

Let vertex C be$(\beta,0)$

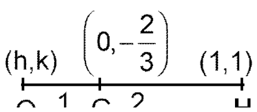
Line AH ⊥ BC

$m_{AH}.m_{BC} = -1$

$$\frac{1}{2}\cdot\frac{2}{\beta-1} = -1 \Rightarrow \beta = 0$$

Centroid of ΔABC is $\left(0,-\frac{2}{3}\right)$

Now G(centroid) divides line joining circum centre (O) and ortho centre (H) in the ratio 1: 2

⇒ 

$2h + 1 = 0 \qquad 2k + 1 = -z$

$h = -\frac{1}{2} \qquad k = -\frac{3}{2}$

$\Rightarrow$ circum centre is $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

Equation of circum circle is (passing through C(0,0)) is

$x^2 + y^2 + x + 3y = 0$

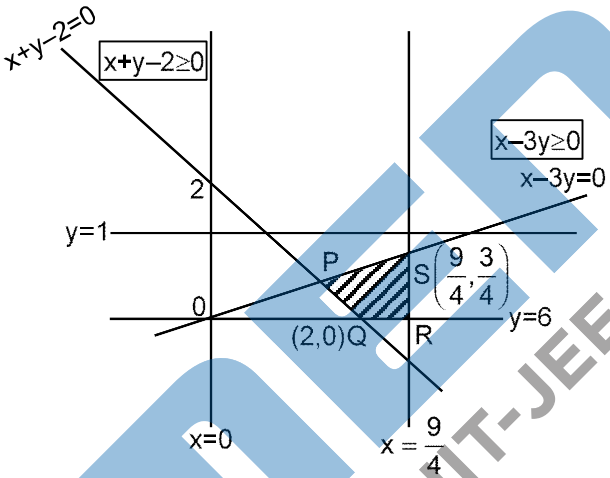
2. The area of the region $\left\{(x, y) : 0 \le x \le \frac{9}{4}, 0 \le y \le 1, x \ge 3y, x + y \ge 2\right\}$ is

(A) $\frac{11}{32}$ (B) $\frac{35}{96}$ (C) $\frac{37}{96}$ (D) $\frac{13}{32}$

Ans. (A)

Sol. $x + y - 2 = 0$

$P\left(\frac{3}{2}, \frac{1}{2}\right)$; Q(2,0); $R\left(\frac{9}{4}, 0\right)$; $S\left(\frac{9}{4}, \frac{3}{4}\right)$



$$\text{Area} = \frac{1}{2}\left[\begin{vmatrix} \frac{3}{2} & \frac{1}{2} \\ 2 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ \frac{9}{4} & 0 \end{vmatrix} + \begin{vmatrix} \frac{9}{4} & 0 \\ \frac{9}{4} & \frac{3}{4} \end{vmatrix} + \begin{vmatrix} \frac{9}{4} & \frac{3}{4} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix}\right]$$

$$= \frac{1}{2}\left|(0 - 1) + (0 - 0) + \left(\frac{27}{16} - 0\right) + \left(\frac{9}{8} - \frac{9}{8}\right)\right| = \frac{11}{32}$$

3. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements.

Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$.

Then the value of p is

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$

Ans. (A)

Sol. $P = \frac{P(S_1 \cap (E_1 = E_3))}{P(E_1 = E_3)} = \frac{P(B_{1,2})}{P(B)}$

$$P(B) = \underset{\substack{\uparrow \\ \text{If 1,2 chosen at start}}}{P(B_{1,2})} + \underset{\substack{\uparrow \\ \text{If 1,3 chosen at start}}}{P(B_{1,3})} + \underset{\substack{\uparrow \\ \text{If 2,3 chosen at start}}}{P(B_{2,3})}$$

$$P(B_{1,2}) = \frac{1}{3} \times \underbrace{\frac{1 \times {}^3C_1}{{}^4C_2}}_{\text{1 is definitely chosen from } F_2} \times \underbrace{\frac{1}{{}^5C_2}}_{\text{1,2 chosen from } G_2}$$

$$P(B_{1,3}) = \frac{1}{3} \times \underbrace{\frac{1 \times {}^2C_1}{{}^3C_2}}_{\text{1 is definitely chosen from } F_2} \times \underbrace{\frac{1}{{}^5C_2}}_{\text{1,2 chosen from } G_2}$$

$$P(B_{2,3}) = \frac{1}{3} \times \left[\underbrace{\frac{{}^3C_2 \times 1}{{}^4C_2}}_{\text{If 1 is not chosen from } F_2} \times \frac{1}{{}^4C_2} + \underbrace{\frac{1 \times {}^3C_1}{{}^4C_2}}_{\text{If 1 is chosen from } F_2} \times \frac{1}{{}^5C_2} \right]$$

$$\frac{P(B_{1,2})}{P(B)} = \frac{1}{5}$$

4. Let $\theta_1, \theta_2, \ldots, \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + \ldots + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}$, $z_k = z_{k-1}e^{i\theta_k}$ for $k = 2, 3, \ldots, 10$, where $i = \sqrt{-1}$. Consider the statements P and Q given below :

$P : |z_2 - z_1| + |z_3 - z_2| + \ldots + |z_{10} - z_9| + |z_1 - z_{10}| \le 2\pi$

$Q : |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \le 4\pi$

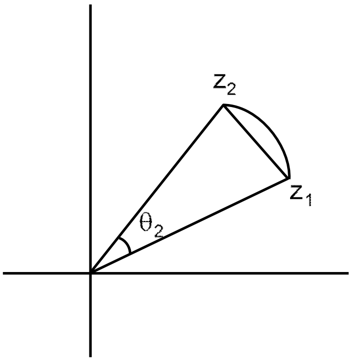
Then,

(A) P is TRUE and Q is FALSE (B) Q is TRUE and P is FALSE

(C) both P and Q are TRUE (D) both P and Q are FALSE

Ans. (C)

Sol.



$|z_1| = |z_2| = \ldots |z_{10}| = 1$

$\text{angle} = \dfrac{\text{arc}}{\text{rad}}$

$\theta_2 = \text{arc}(z_1z_2) > (z_2 > z_1)$

$P : |z_2 - z_1| + \ldots + |z_1 - z_{10}| \le \theta_1 + \theta_2 + \ldots + \theta 10$

$\Rightarrow |z_2 - z_1| + \ldots + |z_1 - z_{10}| \le 2\pi$ P is true

$z_1^2 = e^{i2\theta_1},\ z_k^2 = z_{k-1}^2 \cdot e^{i2\theta_k}$

Let $2\theta_k = \alpha k$

$z_1^2 = e^{i\alpha_1},\ z_k^2 = z_{k-1}^2 \cdot e^{i\alpha_k}$

$\alpha_1 + \alpha_2 + \ldots + \alpha_k = 4\pi$

one similar sense

$\left|z_1^2 - z_2^2\right| + \ldots + \left|z_1^2 - z_{10}^2\right| \le 4\pi$

Q is also true

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;

Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 5 and 6

Question Stem

Three numbers are chosen at random, one after another with replacement, from the set S = {1, 2, 3, ..., 100}. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

5. The value of $\frac{625}{4}p_1$ is __________.

Ans. (76.25)

Sol. p_1 = probability that maximum of chosen numbers is at least 81

p_1 = 1 – probability that maximum of chosen number is at most 80

$$p_1 = 1 - \frac{80 \times 80 \times 80}{100 \times 100 \times 100} = 1 - \frac{64}{125}$$

$$p_1 = \frac{61}{125}$$

$$\frac{625p_1}{4} = \frac{625}{4} \times \frac{61}{125} = \frac{305}{4} = 76.25$$

the value of $\frac{625p_1}{4}$ is 76.25

6. The value of $\frac{125}{4}p_2$ is __________.

Ans. (24.50)

Sol. p_2 = probability that minimum of chosen numbers is at most 40

= 1 – probability that minimum of chosen numbers is at least 41

$$= 1 - \left(\frac{600}{100}\right)^3$$

$$= 1 - \frac{27}{125} = \frac{98}{125}$$

$$\frac{125}{4}p_2 = \frac{125}{4} \times \frac{98}{125} = 24.50$$

Question Stem for Question Nos. 7 and 8

Question Stem

Let α, β and γ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$

$$4x + 5y + 6z = \beta$$

$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let |M| represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point (0, 1, 0) from the plane P.

7. The value of |M| is __________.

Ans. (1.00)

8. The value of D is __________.

Ans. (1.50)

Sol. $7x + 8y + 9z - (\gamma - 1) = A(4x + 5y + 6z - \beta) + B(x + 2y + 3z - \alpha)$

$x : 7 = 4A + B$

$y : 8 = 5A + 2B$

$A = 2, B = -1$

const. term : $-(\gamma - 1) = -A\beta - \alpha B \Rightarrow -(\gamma - 1) = 2\beta + \alpha$

$\alpha - 2\beta + \gamma = 1$

$$M = \begin{vmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \alpha - 2\beta + \gamma = 1$$

Plane P : $x - 2y + z = 1$

Perpendicular distance $= \left|\frac{3}{\sqrt{6}}\right| = P \Rightarrow D = P^2 = \frac{9}{6} = 1.5$

Question Stem for Question Nos. 9 and 10

Question Stem

Consider the lines L_1 and L_2 defined by

$L_1 : x\sqrt{2} + y - 1 = 0$ and $L_2 : x\sqrt{2} - y + 1 = 0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the square of the distance between R' and S'.

9. The value of λ^2 is __________.

Ans. (9.00)

Sol. $P(x,y)\quad \left|\dfrac{\sqrt{2}x+y-1}{\sqrt{3}}\right|\left|\dfrac{\sqrt{2}x-y+1}{\sqrt{3}}\right|=\lambda^2$

$$\left|\frac{2x^2+(y-1)^2}{3}\right|=\lambda^2,\ C:\left|2x^2-(y-1)^2\right|=3\lambda^2$$

line y = 2x + 1, $RS=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$, $R(x_1,y_1)$ and $S(x_2,y_2)$

$y_1 = 2x_1 + 1$ and $y_2 = 2x_2 + 1 \Rightarrow (y_1 - y_2) = 2(x_1 - x_2)$

$$RS=\sqrt{5(x_1-x_2)^2}=\sqrt{5}\left|x_1-x_2\right|$$

solve curve C and line y = 2x + 1 we get

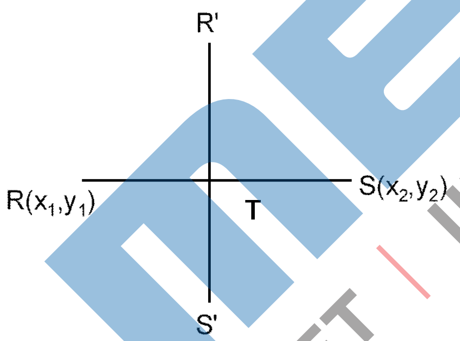
$$\left|2x^2-(2x)^2\right|=3\lambda^2\Rightarrow x^2=\frac{3\lambda^2}{2}$$

$$RS=\sqrt{5}\left|\frac{2\sqrt{3}\lambda}{\sqrt{2}}\right|=\sqrt{30}\lambda=\sqrt{270}\Rightarrow 30\lambda^2=270\Rightarrow\lambda^2=9$$

10. The value of D is __________.

Ans. (77.14)

Sol.



$\perp$ bisector of RS

$$T\equiv\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Here $x_1 + x_2 = 0$

T = (0,1)

Equation of

$$R'S':(y-1)=-\frac{1}{2}(x-0)\Rightarrow x+2y=2$$

$R'(a_1,b_1)$ $S'(a_2,b_2)$

$D = (a_1 - a_2)^2 + (b_1 - b_2)^2 = 5(b_1 - b_2)^2$

solve $x + 2y = 2$ and $\left|2x^2 - (y-1)^2\right| = 3\lambda^2$

$$\left|8(y-1)^2 - (y-1)^2\right| = 3\lambda^2 \Rightarrow (y-1)^3 = \left(\frac{\sqrt{3}\lambda}{\sqrt{7}}\right)^2$$

$$y - 1 = \pm\frac{\sqrt{3}\lambda}{\sqrt{7}} \Rightarrow b_1 = 1 + \frac{\sqrt{3}\lambda}{\sqrt{7}},\ b_2 = 1 - \frac{\sqrt{3}\lambda}{\sqrt{17}}$$

$$D = 5\left(\frac{2\sqrt{3}\lambda}{\sqrt{7}}\right)^2 = \frac{5\times4\times3\lambda^2}{7} = \frac{5\times4\times27}{7} = 77.14$$

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
- Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
- Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
- Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
- Zero Marks : 0 If unanswered;
- Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

  choosing ONLY (B) will get +1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

  choosing any other option(s) will get −2 marks.

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11. For any 3 × 3 matrix M, let |M| denote the determinant of M. Let

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a nonsingular matrix of order 3 × 3, then which of the following statements is (are) TRUE ?

(A) F = PEP and $p^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

(C) $|(EF)^3| > |EF|^2$

(D) Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

Ans. (A,B,D)

Sol. (A) $PEP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & 2 & 3 \\ 8 & 13 & 18 \\ 2 & 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{pmatrix}$$

$$p^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(B) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

$|E| = 0$ and $|F| = 0$ and $|Q| \neq 0$

$$|EQ| = |E||Q| = 0,\ |PFQ^{-1}| = \frac{|P||F|}{|Q|} = 0$$

$T = EQ + PFQ^{-1}$

$TQ = EQ^2 + PF = EQ^2 + P^2EP = EQ^2 + EP = E(Q^2 + P)$

$|TQ| = |E(Q^2 + P)| \Rightarrow |T|\ |Q| = |E|\ |Q^2 + P| = 0 \Rightarrow |T| = 0$ (as $|Q| \neq 0$)

(C) $|(EF)^3| > |EF|^2$

Here $0 > 0$ (false)

(D) as $P^2 = I \Rightarrow P^{-1} = P$ so $P^{-1}FP = PFP = PPEPP = E$

so $E + P^{-1}FP = E + E = 2E$

$P^{-1}EP + F \Rightarrow PEP + F = 2PEP$

$Tr(2PEP) = 2Tr(PEP) = 2Tr(EPP) = 2Tr(E)$

12. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}.$$

Then which of the following statements is (are) TRUE ?

(A) f is decreasing in the interval (–2,–1)

(B) f is increasing in the interval (1,2)

(C) f is onto

(D) Range of f is $\left[-\frac{3}{2}, 2\right]$

Ans. (A,B)

Sol. $f(x) = \dfrac{x^2 - 3x - 6}{x^2 + 2x + 4}$

$$f(x) = \frac{(x^2 + 2x + 4)(2x - 3) - (x^2 - 3x - 6)(2x + 2)}{(x^2 + 2x + 4)^2}$$

$$f(x)=\frac{5x(x+4)}{\left(x^2+2x+4\right)^2}$$

$f'(x)$: $+$ on $(-\infty,-4)$, $-$ on $(-4,0)$, $+$ on $(0,\infty)$

$$f(-4)=\frac{11}{6},\qquad f(0)=-\frac{3}{2},\qquad \lim_{x\to\pm\infty} f(x)=1$$

Range : $\left[-\frac{3}{2},\frac{11}{6}\right]$, clearly f(x) is into

13. Let E,F and G be three events having probabilities

$P(E)=\frac{1}{8}, P(F)=\frac{1}{6}$ and $P(G)=\frac{1}{4}$ and let $P(E\cap F\cap G)=\frac{1}{10}$.

For any event H, if HC denotes its complement, then which of the following statements is(are) TRUE ?

(A) $P\left(E\cap F\cap G^c\right)\le\frac{1}{40}$ (B) $P\left(E^c\cap F\cap G\right)\le\frac{1}{15}$

(C) $P(E\cup F\cup G)\le\frac{13}{24}$ (D) $P\left(E^c\cap F^c\cap G^c\right)\le\frac{5}{12}$

Ans. (A,B,C)

Sol. $P(E)=\frac{1}{8}; P(F)=\frac{1}{6}; P(G)=\frac{1}{4}; P(E\cap F\cap G)=\frac{1}{10}$

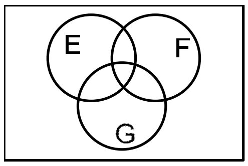
(C) P(E ∪ F ∪ G) = P(E) + P(F) + P(G) – P(E ∩ F) – P(F ∩ G) – P(G ∩ E) + P(E ∩ F ∩ G)

$$=\frac{1}{8}+\frac{1}{6}+\frac{1}{4}-\sum P(E\cap F)+\frac{1}{10}$$

$$=\frac{3+4+6}{24}+\frac{1}{10}-\sum P(E\cap F)=\frac{13}{24}+\frac{1}{10}-\sum P(E\cap F)$$

$\Rightarrow P(E\cup F\cup G)\le\frac{13}{24}$ [(C) is Correct]

(D) $P\left(E^c\cap F^c\cap G^c\right)=1-P(E\cup F\cup G)\ge 1-\frac{13}{24}$



$\Rightarrow P\left(E^c\cap F^c\cap G^c\right)\ge\frac{11}{24}$ [(D) is Incorrect]

(A) $P(E)=\frac{1}{8}\ge P\left(E\cap F\cap G^c\right)+P(E\cap F\cap G)$

$\Rightarrow\frac{1}{8}\ge P\left(E\cap F\cap G^c\right)+\frac{1}{10}\Rightarrow\frac{1}{8}-\frac{1}{10}\ge P\left(E\cap F\cap G^c\right)$

$\Rightarrow\frac{1}{40}\ge P\left(E\cap F\cap G^c\right)$ [(A) is Correct]

(B) $P(F)=\frac{1}{6}\ge P\left(E^c\cap F\cap G\right)+P(E\cap F\cap G)$

$$\Rightarrow \frac{1}{6} - \frac{1}{10} \geq P(E^c \cap F \cap G)$$

$$\Rightarrow \frac{4}{60} \geq P(E^c \cap F \cap G)$$

$$\Rightarrow \frac{1}{15} \geq P(E^C \cap F \cap G) \text{ [(B) is Correct]}$$

14. For any 3 × 3 matrix M, let |M| denote the determinant of M. Let I be the 3 × 3 identity matrix. Let E and F be two 3 × 3 matrices such that (I – EF) is invertible. If G = $(I - EF)^{-1}$, then which of the following statements is (are) TRUE ?

(A) | FE | = | I – FE | | FGE |

(B) | I – FE | (I – FGE) = I

(C) EFG = GEF

(D) (I – FE) (I – FGE) = I

Ans. (A,B,C)

Sol. $|I - EF| \neq 0$; $G = (I - EF)^{-1} \Rightarrow G^{-1} = I - EF$

Now, $G.G^{-1} = I = G^{-1}G$

$\Rightarrow$ G (I – EF) = I = (I – EF) G

$\Rightarrow$ G – GEF = I = G – EFG

$\Rightarrow$ GEF = EFG [C is Correct]

(I – FE) (I + FGE) = I + FGE – FE – FEFGE

= I + FGE – FE – F (G – I) E

= I + FGE – FE – FGE + FE

= I [(B) is Correct]

(So 'D' is Incorrect)

We have

(I – FE) (I +FGE) = I .....(I)

Now

FE(I + FGE)

= FE + FEFGE

= FE + F(G – I)E

= FE + FGE – FE

= FGE

$\Rightarrow$ |FE| |I + FGE| = |FGE|

$$\Rightarrow |FE| \times \frac{1}{|I - FE|} = |FGE| \text{ (from (1))}$$

$\Rightarrow$ |FE| = |I–FE| |FGE|

(option (A) is correct)

15. For any positive integer n, let $S_n : (0, \infty) \to \mathbb{R}$ be defined by

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right),$$

where for any $x \in \mathbb{R}$, $\cot^{-1}x \in (0,\pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then which of the following statements is (are) TRUE ?

(A) $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right)$, for all $x > 0$

(B) $\lim_{x\to\infty} \cot(S_n(x)) = x$, for all $x > 0$

(C) The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0,\infty)$

(D) $\tan(S_n(x)) \le \frac{1}{2}$, for all $n \ge 1$ and $x > 0$

Ans. (A,B)

Sol. $S_n(x) = \sum_{k=1}^{n} \tan^{-1}\left(\frac{x}{1+kx(kx+x)}\right)$

$= \sum_{k=1}^{n} \tan^{-1}\left(\frac{(kx+x)-(kx)}{1+(kx+x)(kx)}\right)$

$S_n(x) = \tan^{-1}(nx+x) - \tan^{-1}x = \tan^{-1}\left(\frac{nx}{1+(n+1)x^2}\right)$

(A) $S_{10}(x) = \tan^{-1}\frac{10x}{1+11x^2} = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right) (x > 0)$

(B) $\lim_{x\to\infty} \cot(S_n(x)) = \lim_{x\to\infty} \frac{\frac{1}{n}+\left(1+\frac{1}{n}\right)x^2}{x} = x\ (x > 0)$

(C) $S_3(x) = \tan^{-1}\frac{3x}{1+4x^2} = \frac{\pi}{4} \Rightarrow 4x^2 - 3x + 1 = 0 \Rightarrow x \notin \mathbb{R}$

(D) $\tan(S_n(x)) = \frac{nx}{1+(n+1)x^2};\ \forall\ n \ge 1;\ x > 0$

We need to check the validity of $\frac{nx}{1+(n+1)x^2} \le \frac{1}{2}\ \ \forall\ n \ge 1\ ;\ x > 0\ ;\ n \in \mathbb{N}$

$\Rightarrow 2nx \le (n + 1)x^2 + 1$

$\Rightarrow (n + 1)x^2 - 2nx + 1 \ge 0\ \forall\ n \ge 1\ ;\ x > 0\ ;\ n \in \mathbb{N}$

Discriminant of $y = (n + 1)x^2 - 2nx + 1$ is

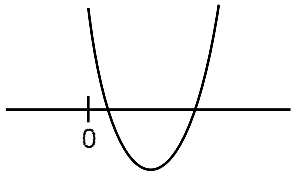
$D = 4n^2 - 4(n + 1)$ and $n \in \mathbb{N}$

$D < 0$ for $n = 1$; true for $x > 0$

$D > 0$ for $n \geq 2 \Rightarrow \exists$ some $x > 0$

for which $y < 0$ as both roots of

$y = 0$ will be positive.



$y = (n + 1)x^2 - 2nx + 1, n \geq 2$

So, $y \geq 0\ \forall\ n \geq 1$; $\forall\ x > 0$; $n \in N$ is false.

16. For any complex number $w = c + id$, let $\arg(w) \in (-\pi,\pi]$, where $i = \sqrt{-1}$. Let α and β be real numbers such that for all complex numbers $z = x + iy$ satisfying $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, the ordered pair (x,y) lies on the circle

$$x^2 + y^2 + 5x - 3y + 4 = 0.$$

Then which of the following statements is (are) TRUE ?

(A) $\alpha = -1$ (B) $\alpha\beta = 4$ (C) $\alpha\beta = -4$ (D) $\beta = 4$

Ans. (B,D)

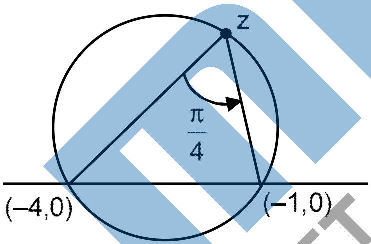
Sol. $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$ implies z is

on arc and $(-\alpha, 0)$ & $(-\beta, 0)$ subtend $\frac{\pi}{4}$ on z.

And z lies on $x^2 + y^2 + 5x - 3y + 4 = 0$

So put $y = 0$;

$x^2 + 5x + 4 = 0 \Rightarrow x = -1$; $x = -4$



Now, $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4} \Rightarrow z + \alpha = (z+\beta)\cdot r\cdot e^{i\frac{\pi}{4}}$

So, $z + \beta = z + 4 \Rightarrow \beta = 4$ & $z + \alpha = z + 1 \Rightarrow \alpha = 1$

SECTION-4 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;

Zero Marks : 0 In all other cases.

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17. For $x \in \mathbb{R}$, then number of real roots of the equation $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is ____.

Ans. (4)

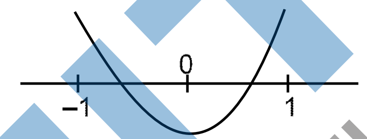
Sol. $3x^2 + x - 1 = 4|x^2 - 1|$

If $x \in [-1, 1]$,

$3x^2 + x - 1 = -4x^2 + 4 \Rightarrow 7x^2 + x - 5 = 0$

say $f(x) = 7x^2 + x - 5$

$f(1) = 3;\ f(-1) = 1;\ f(0) = -1$



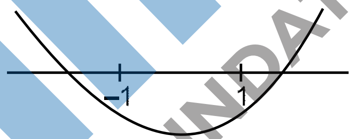
[Two Roots]

If $x \in (-\infty, -1] \cup [1, \infty)$

$3x^2 + x - 1 = 4x^2 - 4 \Rightarrow x^2 - x - 3 = 0$

Say $g(x) = x^2 - x - 3$

$g(-1) = -1;\ g(1) = -3$



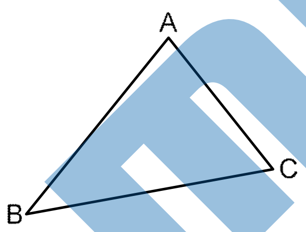
[Two Roots]

So total 4 roots.

18. In a triangle ABC, let $AB = \sqrt{23}$, $BC = 3$ and $CA = 4$. Then the value of $\frac{\cot A + \cot C}{\cot B}$ is_____.

Ans. (2)

Sol.



Given $c = \sqrt{23}$; $a = 3;\ b = 4$

$$\cot A = \frac{\cos A}{\sin A} = \frac{b^2 + c^2 - a^2}{2bc \sin A}$$

$$= \frac{b^2 + c^2 - a^2}{2 \cdot 2\Delta} \left\{ \Delta = \frac{1}{2} bc \sin A \right\}$$

$$\cot A = \frac{b^2 + c^2 - a^2}{4\Delta}$$

Similarly, $\cot B = \frac{a^2 + c^2 - b^2}{4\Delta}$ & $\cot C = \frac{a^2 + b^2 - c^2}{4\Delta}$

$$\therefore \frac{\cot A + \cot C}{\cot B} = \frac{b^2 + c^2 - a^2 + a^2 + b^2 - c^2}{a^2 + c^2 - b^2} = \frac{2b^2}{a^2 + c^2 - b^2} = \frac{32}{16} = 2$$

19. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where u and v are unit vectors which are not perpendicular to each other and $\vec{u}\cdot\vec{w} = 1, \vec{v}\cdot\vec{w} = 1, \vec{w}\cdot\vec{w} = 4$

If the volume of the parallelopiped, whose adjacent sides are represented by the vectors $\vec{u}, \vec{v}$ and $\vec{w}$, is $\sqrt{2}$, then the value of $|3\vec{u} + 5\vec{v}|$ is____.

Ans. (7)

Sol. Given, $|\vec{u}| = 1; |\vec{v}| = 1; \vec{u}\cdot\vec{v} \neq 0; \vec{u}\cdot\vec{w} = 1; \vec{v}\cdot\vec{w} = 1;$

$\vec{w}\cdot\vec{w} = |\vec{w}|^2 = 4 \Rightarrow |\vec{w}| = 2; [\vec{u} \quad \vec{v} \quad \vec{w}] = \sqrt{2}$

and $[\vec{u} \quad \vec{v} \quad \vec{w}]^2 = \begin{vmatrix} \vec{u}\cdot\vec{u} & \vec{u}\cdot\vec{v} & \vec{u}\cdot\vec{w} \\ \vec{v}\cdot\vec{u} & \vec{v}\cdot\vec{v} & \vec{v}\cdot\vec{w} \\ \vec{w}\cdot\vec{u} & \vec{w}\cdot\vec{v} & \vec{w}\cdot\vec{w} \end{vmatrix} = 2$

$= \begin{vmatrix} 1 & \vec{u}\cdot\vec{v} & 1 \\ \vec{u}\cdot\vec{v} & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2$

$\Rightarrow \vec{u}\cdot\vec{v} = \dfrac{1}{2}$

So, $|3\vec{u} + 5\vec{v}| = \sqrt{9|\vec{u}|^2 + 25|\vec{v}|^2 + 2\cdot 3\cdot 5\vec{u}\cdot\vec{v}}$

$= \sqrt{9 + 25 + 30\left(\dfrac{1}{2}\right)} = \sqrt{49} = 7$